SECTION II.—GENERAL METEOROLOGY.

DEFLECTION OF BODIES MOVING FREELY UNDER GRAVITY ON A ROTATING SPHERE.

By CHARLES FREDERICK MARVIN.

[Dated: Weather Bureau, Washington, D. C., Nov. 18, 1915.]

No fundamental principle among the laws of the motions of the atmosphere and ocean currents is perhaps so far reaching in important effects and so difficult for the general student and non-mathematical reader to fully grasp and comprehend, as the deflective influence of the earth's rotation on direction and motion of freely moving bodies. Many expositions of this interesting dynamic problem have been published, and simple explanations are to be found in standard works on meteorology and mechanics. Nevertheless, to many the whole matter is shrouded in obscurity, and those who lack the facility of literally thinking in mathematical equations do not grasp the complete physical reality of the effects or clearly discern the source or character of the actual forces in operation.

In view of the foregoing another explanation and derivation of the deflective forces is now offered that has been developed along the lines of a method already briefly presented in the MONTHLY WEATHER REVIEW.[1] That article, however, presupposes a good deal of knowledge of the subject on the part of the reader and, moreover, fails to indicate clearly the real nature and origin of the deflecting forces and how they act, all of which the present explanation aims to supply. While lacking all the elegance, generality, and directness of the established mathematical demonstrations, the new presentation seeks to visualize and make obvious the several forces, conditions, and effects in a manner that the general student with just the ordinary familiarity with the principles of mechanics and the parallelogram of forces will readily follow and clearly comprehend.

In order that the reader may be better prepared to follow what is presented later it seems proper to say at this point that air in motion, flowing rivers, moving trains, any objects whatsoever moving over the surface of the earth are subjected to a greater or less deflective influence that tends to cause the body to swerve to the right in the Northern Hemisphere and to the left on its onward course in the Southern Hemisphere. In ordinary language these results are said to be caused by the earth's rotation. In fact, however, the deflections are caused by components of gravity which act in this manner whenever any body is set in motion relative to the rotating earth; the moving earth is merely a condition—an essential to the situation. Moreover, the effects are not imaginary or illusive, but are real and due to the action of real forces. Gravity, acting by means of components, is the real source of the deflective influences. When a body not on the Equator moves exactly eastward or westward the deflecting force is a real, lateral component of gravity. In this case the speed of the moving body is not affected in the slightest degree. The *direction of motion* only is changed. For motion exactly northward or southward, however, just the reverse is true. A component of gravity then acts on air moving *toward the pole* to *increase* its eastward velocity and vice versa for air moving equatorward. The deflection toward the east or west, of air moving on the meridian is therefore more apparent than real, in that the deflection results from a change in the actual as well as in the relative velocity of the body in question, and at the start is not literally a change of direction of motion. The effect, nevertheless, is due to a real component of the force of gravity, as will hereafter be shown. As soon as the deflected current acquires any appreciable component of motion eastward or westward the true lateral component of gravity then also acts literally to change the direction of motion. The following will give an idea of the magnitude of these effects: If a ball could be set in motion in a frictionless manner in any direction whatever on a smooth horizontal surface at latitude 40° and at a velocity of 1 mile an hour, the ball would travel in very nearly a closed circle of radius of a little less than 3 miles. Each successive curve thus described would fall a trifle west of its predecessor; and the motion in the circle would be right-handed in the Northern Hemisphere, and left-handed in the Southern Hemisphere. If the ball had a velocity of 10 miles per hour the radius would be ten times as great, and so on. The radius will also have different values for different latitudes.

Before matters now under discussion can be properly understood it seems necessary also to comment briefly upon the confusion in the use of the expression "centrifugal force." This term runs all through studies and discussions of the paths of moving bodies and rotating objects, and prominent writers are sometimes careless in the use of the word, with the result that the beginner and nontechnical reader are hopelessly confused. "Centrifugal force" is not a real force; that is, if the word "force" thus used has the same meaning it has when ordinarily applied to other of the recognized forces.

Whenever any body whatsoever moves along a curved path, some deflective force is always acting perpendicular to the path and directly towards the center of curvature. If it were not for such a radial or centripetal force the body would move in a straight line. A body moving in a curved path always acts *as if* it *tended* to fly away from the center and writers often speak of this tendency as "centrifugal force." It must be distinctly recognized, however, that the "force" or tendency acting *away* from the center is simply an inertia reaction of the body and that there is, in fact, a radial or centripetal force acting on the body to constantly deflect it *toward* the center and away from the straight line it is momentarily following.

Attention having been called to certain sources of confusion and the character of the phenomena under consideration clearly indicated, the real problem before us may now be taken up.

[1] *T. Okada*, in MONTHLY WEATHER REVIEW, May, 1908, 36: 147.

Let *NESW* in figure 1 be a meridional section of the geoidal figure of the earth with excentricity greatly exaggerated. Let ω be the angular velocity of rotation about the axis *NS*. Imagine a body somehow held at rest at *P*, that is, held so as not to partake of the earth's rotation. The geocentric attraction of the earth for such a body will draw it directly towards the center of mass *C*, and this force may be represented in magnitude and direction by the line *PO* through *C*. If, however, the body at *P* is rotating *with* the earth it will move in a circular path corresponding to the parallel of latitude of its place and must, therefore, be acted upon by a radial centripetal force as *PF*. This force is supplied by the gravitational attraction of the earth, and is, therefore, a component of *PO*, readily shown as such by completing the parallelogram of forces *FOO'P*. The apparent force of gravity at *P* on the rotating earth is, therefore, less than the true geocentric attraction (*PO*) and is shown in amount and direction by the line *PO'*. The force *PO'* lies in the direction of the plumb line at *P* and is necessarily perpendicular to the geoidal surface at the same point, because by definition the geodial surface is one, each point of which is perpendicular to the plumb line at the corresponding point. The angle $PC'E = \varphi$, is the geographical latitude of the place *P*.

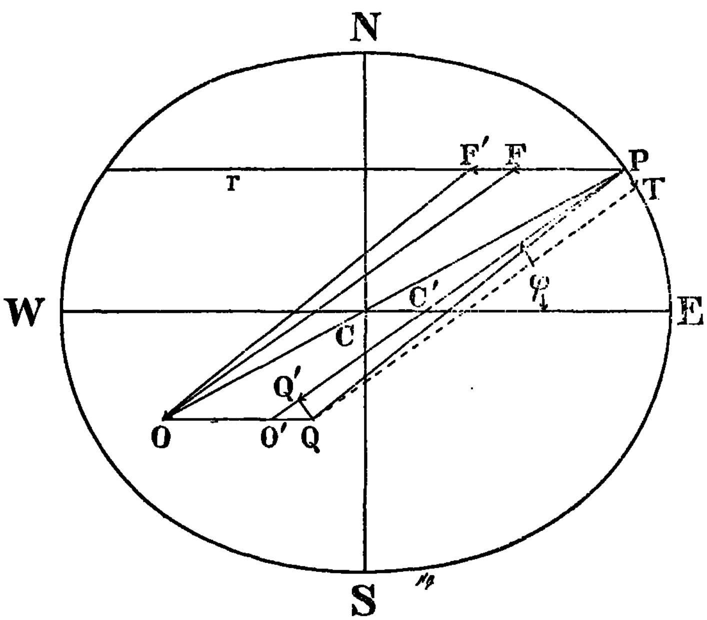

FIG. 1.

Suppose, now, the body or mass, *m*, at *P* moves eastward over the earth at a uniform velocity, *v*, a greater radial force than *PF* now becomes necessary to compel the body to remain on the surface of the earth. Let *PF'* represent the new force which, as before, is a component of the central gravitational attraction of the earth. The new gravitational forces acting on the body (now moving eastward) are found by completing the parallelogram *F'OQP*. The component *PQ* is now the apparent force of gravity for a body at *P moving eastward* at velocity *v*. This force, however, is no longer perpendicular to the geoidal surface; hence a component of force to cause horizontal motion arises, so that by resolution of forces we get the attraction *PQ'* strictly perpendicular, and the tangential component *PT*, which is the sought-for deflective force. As the body is supposed to be moving eastward (perpendicular to the plane of the picture in this case) it is plain *PT* acts to deflect the motion to the right. If the body moves from east to west, the radial force *PF'* must be less than *PF*, and it is easy to see this would cause the point corresponding to *T* to fall on the other side of *P*, and the body would be deflected polewards but still to the right of its onward course.

It is interesting to notice that, while *PO'* represents the attraction of gravity on a stationary mass of air at *P*, the attraction for and hence the barometric pressure of this same air mass moving eastward at velocity *v* is less, and is represented by *PQ'*.

The formula for stating the value of *PT* may be found as follows:

Let $PC'E = \varphi$ = the geographic latitude of *P*.
ω = angular velocity of earth's rotation
= velocity of particle at unit distance from axis.
v = linear velocity, west to east, of body at *P*.
r = distance from surface to axis of earth at latitude φ.
f_e = deflective force for east-west motion,
= $QQ' = PT$.

The actual angular velocity of a body moving eastward at *P* with velocity *v* is

$$\omega + \frac{v}{r}$$

(If the body moves westward $\frac{v}{r}$ will be negative.)

Therefore the radial force, *PF'*, necessary to maintain motion in a circle is

$$PF' = m\left(\omega + \frac{v}{r}\right)^2 r = OQ,$$

also

$$PF = m\omega^2 r = OO',$$
$$FF' = PF' - PF = O'Q,$$

Therefore for the deflective force *QQ'* we have

$$f_e = QQ' = O'Q \sin\varphi = m\left[\left(\omega + \frac{v}{r}\right)^2 r - \omega^2 r\right] \sin\varphi,$$

that is,

$$f_e = m\,r\left(\frac{2\omega v}{r} + \frac{v^2}{r^2}\right)\sin\varphi.$$

Except very near the poles, or for very high values of *v*, such as might occur, for example, in studies of the motions of projectiles, the term $\frac{v^2}{r^2}$ will be so small that it may be neglected, in which case the deflective force becomes

$$f_e = 2\,m\omega v \sin\phi$$

Since, as was pointed out above, the attraction of gravity on a moving mody is not only less than when the body is at rest on the earth, but is also inclined to the plumb line at the same place, it may be pointed out as a direct corollary that the mean surface of a moving river is not strictly horizontal. The angular deviation from a horizontal surface is represented in the diagram by the angle *QPQ'*.

DEFLECTION FOR MOTION NORTH AND SOUTH.

To understand how gravity accomplishes this result it is necessary to be familiar with a fundamental principle of the dynamics of bodies moving freely under attractive forces like gravity. This principle is the law of equal areas or the constancy of angular momentum. It is fully explained in elementary textbooks of astronomy, and when applied to the case of air moving over the rotating surface of the earth this law requires that the square of the distance from the axis of rotation multiplied by the angular velocity must be a constant, so that if r is the distance from the axis and ω the angular velocity, then $r^2\omega$ = constant.

This product is also called the angular momentum. The upper portion of figure 2 represents a meridional elevation of the Northern Hemisphere. Below is shown one-half of a circumpolar projection of the same region. Let us suppose that a body at rest on the earth at A will be carried to A' in a short time, t seconds, by reason of

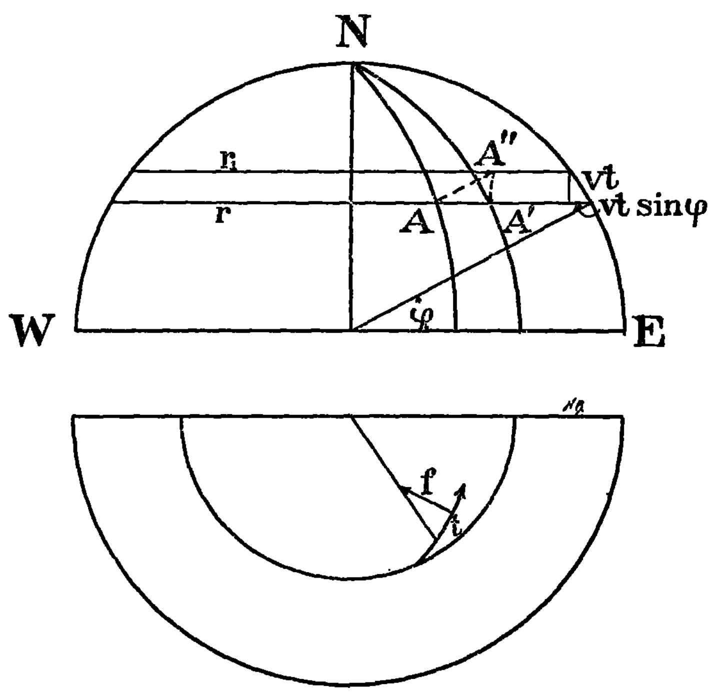

FIG. 2.

the earth's rotation. Let ω indicate the angular velocity of this motion. If now the body at A is moving northward at the uniform velocity V it will not be found at A' at the end of time, t, but will occupy a position as at A''. In this position it is nearer the axis of the earth than before, that is, r_1 is smaller than r and the angular velocity of the body must be greater, because the law of equal areas requires that

$$r^2\omega = r_1^2\omega_1, \qquad \text{or} \qquad \omega_1 = \frac{\omega r^2}{r_1^2}.$$

The gain in angular velocity in t seconds is, therefore,

$$\omega_1 - \omega = \frac{\omega r^2}{r_1^2} - \omega.$$

Strictly speaking the rate of gain in velocity is not necessarily the same from instant to instant when the changes of latitude of the moving body are considerable, nevertheless for all meteorological purposes we may safely neglect secondary variations; that is, assume that the rate of change in angular velocity is constant. Therefore, the gain in velocity in one second is $\frac{\omega_1-\omega}{t}$. Now the gain in velocity of a body in one second is commonly called "acceleration" and may be represented in the present case by a. Moreover since the surface velocity in the position A'' equals r_1 times the angular velocity we may write

$$\frac{(\omega_1-\omega)r_1}{t} = a = \omega\left(\frac{r^2-r_1^2}{tr_1^2}\right)r_1,$$

$$a = \omega\,\frac{(r+r_1)\;(r-r_1)}{tr_1}.$$

Now for the motions taking place in a small interval of time like one second, and except for places quite near the pole, r and r_1 are very nearly equal, so that $\frac{r+r_1}{r_1}=2$.

Referring to the portion of the upper figure at the right-hand side, it is plain that when t is a relatively small time the difference between the two radii r_1 and r is given by the equation $r-r_1=vt\sin\varphi$. From these values we get

$$a = 2\omega v \sin\varphi.$$

A body moving steadily northward is, therefore, found to gain eastward velocity at the rate shown by the above equation. To an observer unconscious of the earth's rotation the body seems to move from A' to A'' as if it were deflected eastward by some deflecting force. Since it is customary to measure forces by the product of mass by acceleration we can write in this case

$$f_n = ma = 2m\omega v \sin\varphi.$$

From this we find the deflective effect in the case of north-south motion is of exactly the same numerical value as in the case of east-west motion.

It will doubtless be admitted from the foregoing, without further demonstration, that any body moving in any direction over the surface of a rotating sphere seems to be deflected by a force of the magnitude given in the following equation

$$f = 2m\omega v \sin\varphi.$$

As the proof of this brings out a point of some interest this also will be briefly given.

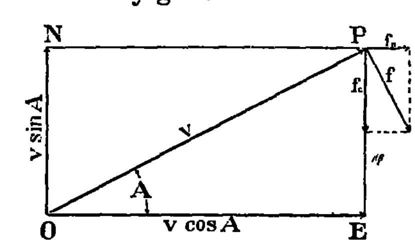

FIG. 3.

In figure 3 let OP represent the velocity and direction of motion of some body. This motion can be regarded as compounded of an eastward motion OE and a northward motion ON. Let v equal the velocity of motion along OP; then

Northward velocity $= ON = v\sin A$,
Eastward velocity $= OE = v\cos A$.

From the previous demonstration we have

$$f_n = 2m\omega v \sin A \sin \varphi.$$
$$f_e = 2m\omega v \cos A \sin \varphi.$$

These deflective forces are shown in the diagram, also their resultant deflection f, the value of which must be

$$f = \frac{f_n}{\sin A} = 2m\omega v \sin \varphi.$$

$$\text{or} \qquad f = \frac{f_e}{\cos A} = 2m\omega v \sin \varphi. \tag{1}$$

Viewed in this manner the deflective influence on any moving body may be regarded as compounded of an effect for motion in the meridian, which we have found accelerates or retards the angular velocity of the moving body, and another effect for motion on a parallel of latitude which changes the direction of motion without affecting velocity.

A little examination will show that the deflective action must be to the right in the Northern and to the left in the Southern Hemisphere.

How gravity acts to accelerate the eastward motion of the body is clearly shown in the lower part of figure 2. As the body moves northward it moves spirally inward towards the axis of the earth. The action of gravity in this direction is inclined to the path and one component of this force, t, accelerates the motion, while the other, f, neutralizes the "centrifugal" tendency, that is, keeps the body from flying off on a tangent.

To compute numerical values of forces by equation (1) the proper units must be employed. There is practically only one unit of force in scientific use, namely, the dyne. In order that the equation may give the force in dynes, it is necessary simply that v be measured in centimeters per second and that the angular velocity of the earth, ω, be expressed in angular amount of motion per second. The earth makes a complete rotation in one siderial day, that is, in 86,164 ordinary seconds. Since the whole circumference is 2π,

$$\omega = \frac{2\pi}{86{,}164}.$$

Since high wind velocities expressed in centimeters per second are inconveniently large numbers, some may prefer to express v in miles per hour. The equation then becomes:

$$f_{\text{dynes}} = 0.006520m\, v_m \sin \varphi.$$

CAUSE OF "SMOKE" FROM MOUNT HOOD.

By FLOYD D. YOUNG, Assistant Observer.

[Dated: Weather Bureau, Portland, Oreg., Nov. 9, 1915.]

The Portland "Oregonian" of October 21, 1915, published the following:

MOUNT HOOD SEEN TO SMOKE—PECULIAR PHENOMENON OBSERVED BY RESIDENTS OF THE DALLES.

(Special.)

THE DALLES, OREG., *October 20.*

Groups of citizens here this afternoon, about 5 o'clock, watched for many minutes a circle of smoke which appeared to be issuing from the vicinity of the crater of Mount Hood, 1,000 feet below the summit.

It was smoke from the crater or a most peculiar atmospheric condition never before seen at that point on the mountain. Before nightfall all the town was interested, as the stories of the various groups of witnesses became noised about. Many old residents were among the watchers.

This is the latest of a long series of such reports sent in at intervals from various towns in the vicinity of Mount Hood; hardly a year of the last decade has passed without the so-called smoke being seen from some locality. This report, like its predecessors, was telegraphed by the news syndicates to all parts of the United States and like its predecessors, it evoked a great deal of discussion and denial from geologists. Probably this latest story gained more credence than the ones of former years, on account of the late activity of Mount Lassen in California.

Geologists unite in saying that the mountain has not been active since the earth became inhabited and that the danger of an eruption at this time is inconsiderable. Instead of being a potential source of evil, a dormant volcano ready at any time to burst forth and overwhelm the surrounding country, Mount Hood is really one of the great natural resources of the State of Oregon. It acts as a great reservoir, storing up water in the form of snow all through the winter and gradually giving it up through the dry summer months, so that the water stages of the four rivers that have their origin at its base vary but little throughout the year. In the Hood River Valley there is a saying, "The warmer the day, the more water for irrigation." The great glacier that brings Hood River into being advances down the mountain side during the winter months and gradually retreats before the summer sun. On some summer days the temperature at the foot of this ice field reaches 85°F. and over, and the rapid melting of the ice is made evident by the continual clatter of the bowlders as they roll down the long steep terminal moraine. Were it not for the melting snows of Mount Hood many streams now used for water power would be practically dry in summer and the city of Portland, Oreg., would be forced to look elsewhere for its water supply.

There is no doubt that the observers who saw the mountain "smoking" were sincere in their statements, for the phenomenon is not unusual. There are three kinds of "smoke" that rise from the top of Mount Hood and all three are dependent on meteorological conditions and on the peculiar topography of the mountain top. Evidently at one time a high inward-facing cliff extended in a complete circle around the summit, forming the walls of the crater. The southern and eastern segments of this circle have disappeared, leaving a semicircular wall of rock facing west and north. (See figs. 1 and 2.) When loose dry snow is present below the cliffs inside the crater rim, a strong wind blowing from the south or east will sweep it to the middle of the arc and there throw it high into the air.

A case of this kind was witnessed by the writer from Portland on October 25, 1915. The sun was rather low in the west and was locally obscured by clouds, but it shone full on the mountain and brought it out in bold relief against the sky. A thin plume of white was rising from the top of the peak to a considerable height, and then curling off toward the north. It was so plain as to be unmistakable, wavering in the wind, now rising, now falling, until it finally disappeared entirely. From the time it was first seen until it disappeared was about four minutes, but it is not known how long the "smoke" had been rising before it was seen. That the wind was blowing from the south was shown by the movement of a few small clouds east of the mountain top and at about the altitude of the summit.

The second type of "smoke" is explained by the great variety of cloud forms that are seen around the mountain at different times. Small clouds have often been seen to drift into the open side of the crater ring and be transformed into an almost perpendicular column of vapor by